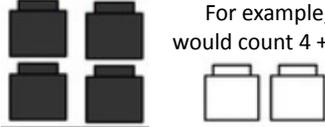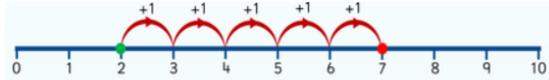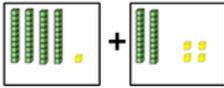# Mathematics Written Calculation Progression Document

These are the methods we teach at school in the order they are taught. Note that each time a new digit is added, it may be beneficial to return to concrete objects to consolidate understanding. It is also useful for children to have access to a variety of methods and to understand that they can solve problems in different ways. Across all stages, physical objects, drawings and models should be used to consolidate understanding.

Addition: By the time children leave the school, they should be able to use column method addition confidently for large numbers.

The different stages	Example of what it looks like	The different stages	Example of what it looks like
Stage 1 Counting sets of objects	For example, counting cubes. Also use songs to consolidate.	Stage 6 Using concrete objects, such as Deines or Cuisenaire to show and calculate adding tens together and adding ones together.	$41 + 24 = 65$ You could use a place value chart alongside objects at this stage.
Stage 2 Combining 2 sets of objects into one group and counting practically	For example, using cubes children would count $4 + 2 = 6$	Stage 7 Partitioned numbers are written underneath one another.	$47 + 76 = 123$ $47 =$ $+ 76$ $40 + 7$ $70 + 6$ $110 + 13 = 123$
Stage 3 Drawing pictures/dots – informal jottings. Then counting how many altogether.	$4 + 2 = 6$ **** + ** = ******	Stage 8 Numbers are written into columns. Often tens are added first.	47 $+ 76$ 110 $\underline{13}$ 123
Stage 4 Counting on, on a number line with numbers on it.	$2 + 5 = 7$ +1 +1 +1 +1 +1	Stage 9 Numbers are written into columns. Ones added first.	47 $+ 76$ 13 $\underline{110}$ $\underline{123}$
Stage 5a Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 1) Partition the smaller number into tens and ones 2) Add on the tens 3) Add on the units	$8 + 7 = 15$	Stage 10 This becomes the shorter method and numbers get carried to the next	47 $+ 76$ $\underline{123}$ 1
		Stage 11 Move to adding three two-digit numbers and numbers with different numbers of digits.	176 $\underline{+ 147}$ $\underline{323}$ $1\ 1$ 28 47 $\underline{+\ 76}$ $\underline{151}$ 12

	  $48 + 36 = 84$ partitioning 30 6

		This is the National Curriculum formal written method.	
Stage 5b Using a bar-model to demonstrate part-part whole and for solving missing number problems.	$25 + 75 = 100$ Note: Number lines can also be used for solving missing number problems and understanding inverse operations.	Stage 12 Move on to add numbers with 4 digits and more including decimals.	

Subtraction: By the time children leave the school, they should be able to use column method subtraction confidently for large numbers.

The different stages	Example of what it looks like	The different stages	Example of what it looks like
Stage 1 Using a set of objects and practically taking some away.	For example, using cubes or balls. 	Stage 6 Partitioned numbers are written underneath each other in columns of tens and ones and later by hundreds, tens and ones and so on. This is how we introduce place value and column subtraction.	$74 - 23 =$ $\begin{array}{r} 7\ 0 + 4 \\ -\ 2\ 0 + 3 \\ \hline 5\ 0 + 1 \quad = 5\ 1 \end{array}$
Stage 2 Using informal jottings to subtract by crossing out.	$7 - 2 = 5$ * * * * * | | *		
Stage 3 Count back on a number line with numbers on	 $8 - 3 = 5$		
Stage 4a Using a number line a) Working by counting backwards in chunks b) Finding the difference by counting on in chunks c) Solving missing number problems	$15 - 13 = 2$ $72 - 56 = 16$ Or find the difference between 56 and 72 $13 +$ ▮ $= 18$	Stage 7 (exchanging – 2-digit numbers) 1) Partition number into tens and ones 2) Always start with the ones column 3) Exchange where necessary	$74 - 27=$ $\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array}$ $\begin{array}{r} {}^{60}\ {}^{14} \\ \cancel{70}+\cancel{4} \\ -\ 20+7 \\ \hline 40+7 \end{array}$ ➡ $\begin{array}{r} {}^{6}\ {}^{14} \\ \cancel{7}\,\cancel{4} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$

Stage 4b Using a bar-model to demonstrate part-part whole	$100 - 25 = 75$ 	100
	25		Stage 8 (exchanging 3-digit numbers)	$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array}$ $\begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array}$ $\Rightarrow$ $\begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ 1 \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$
Stage 5 Concrete objects, such as Deines, Numicon and Cuisenaire are used to show and calculate T0 – T0 It is useful to use alongside place value charts here.	$64 - 14 = 50$ $32 - 20 = 12$	Stage 9 Full column addition for any number including numbers where exchanging involves zeroes. **This is the National Curriculum formal written method.**	$\begin{array}{r} 4000\ \ 900\ \ 100 \\ \cancel{5000}+\cancel{000}+\cancel{00}+8 \\ -\ 1000+200+50+7 \\ \hline 3000+700+50+1 \end{array}$ $\Rightarrow$ $\begin{array}{r} \overset{9}{}\ \ \\ 4\ \ \overset{10}{\cancel{0}}\ \overset{10}{\cancel{0}}\ \ \\ 5\ 0\ 0\ 8 \\ -\ 1\ 2\ 5\ 7 \\ \hline 3\ 7\ 5\ 1 \end{array}$

Multiplication

In addition to written methods of multiplication, children should be able to: Phase 1: derive and recall multiplication facts (and corresponding division facts) for 2, 5- and 10-times tables Phase 2 Year 3: derive and recall multiplication facts (and corresponding division facts) for 3, 4- and 8-times tables Phase 2 Year 4: derive and recall multiplication facts (and corresponding division facts) for all times tables up to 12x12 Phase 3: Use multiplication and corresponding division facts for problem solving.		*The different stages*	*Example of what it looks like*
		Stage 6 **The grid method** It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the answers to each part of the multiplication together.	$7 \times 38 =$ $\begin{array}{c|c} \times & 7 \\ \hline 30 & 210 \\ 8 & 56 \\ \hline & 266 \end{array}$
The different stages	*Example of what it looks like*	Stage 7 **Expanded method** Showing method and stages of working using a column format. This stage links stages 5 and 6 with the formal column method.	$\begin{array}{r} 30+8 \\ \times\ \ \ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$ $\begin{array}{l} 30 \times 7 = 210 \\ 8 \times 7 =\ \ 56 \end{array}$ $\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$ This is the slightly shorter version.
Stage 1 Counting practically in repeated groups/patterns	Twos: 2, 4, 6 Fives: 5, 10, 15 Tens: 10, 20, 30		
Stage 2 Grouping	$2 \times 3 = 6$		
Stage 3a **Repeated addition with objects and drawings**	$5 \times 3 = 5+5+5 = 15$ Example with Numicon	Stage 8 **Short multiplication for T0 x 0**	$38 \times 7 =$ 38
Stage 3b **Repeated addition**			

5 times three is 5+5+5 or 3 lots of 5 or 5x3 Repeated addition can be shown easily on a number line Arrays	5 5 5 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 5 10 15	Remainders are added mentally and not all steps are recorded.	X 7 266 5

Stage 4
Arrays
	3 x 2 = 6 Or 2 x 3 = 6	Stage 9
		Short multiplication from HT0 x 0 to ThHT0 x 0
		
		This is the National Curriculum formal written method.
			342 x 7 becomes
			
			  3 4 2
			x 7
			2 3 9 4
			 2 1
			
			Answer: 2394
				2741 x 6 becomes
				
				  2 7 4 1
				x 6
				1 6 4 4 6
				 4 2
				
				Answer: 16, 446

Stage 5
Partitioning
	36 x 7 = 30 x 7 + 6 x 7
	     = 210 + 42
	     = 252			

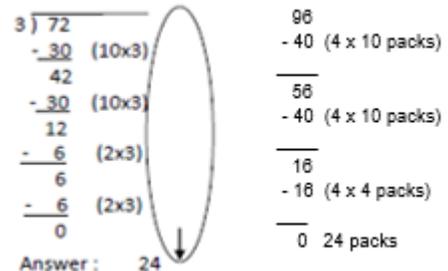|	
Stage 10 **Grid method for TO x TO**	74 x 23 = | x	20	3	Totals
---	---	---	---
70	1400	210	1610
4	80	12	92
			1702	 1 You can add vertically or horizontally to find the answer.

Stage 11
The amount of recording is reduced but children follow grid method steps.
	56 × 27 is approximately 60 × 30 = 1800.
	
	   56
	× 27
	1000 50 × 20 = 1000
	 120 6 × 20 = 120
	 350 50 × 7 = 350
	  42 6 × 7 = 42
	1512
	  1

Stage 12
Full column method
 	
Extend to HT0 x T0 then ThHT0 x T0 and including decimals
 	
This is the National Curriculum formal written method.
	56
	X 27
	  392
	1120 +
	1512
	  1 It is suggested that remainders are carried above the numbers rather than below and then crossed out when used to avoid confusion when moving to the next column or to the addition stage.

Division: By the time children leave school, they should confidently be able to use the bus stop method for division, including involving remainders as decimals.
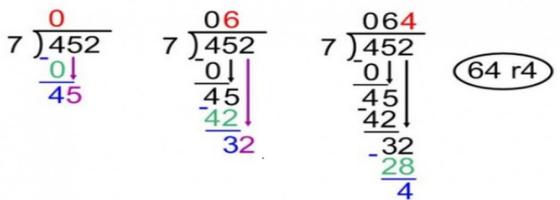
The different stages	Example of what it looks like
Stage 1 Children will develop their understanding of division and use jottings to support calculation.	Sharing equally: 6 sweets shared between 2 people, how many do each get? Grouping or repeated addition: There are 6 sweets, how many people can have 2 sweets each? 10÷2=5
Or alternatively, arrays can be used.	
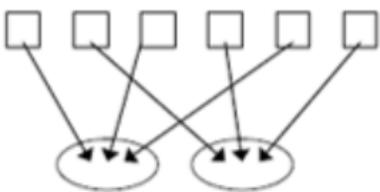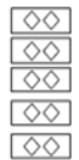
Stage 4 Using multiples of the divisor for repeated addition and subtraction to find answers to division with and without remainders. Initially these will be for 2s, 5s, 10s	 **27÷5=5r2**
Stage 5 Moving onto taking away chunks (known multiples) from the dividend	**27÷5=5r2**
Stage 6 Division with objects TU ÷ U then HTU ÷ U	For example, using counters:
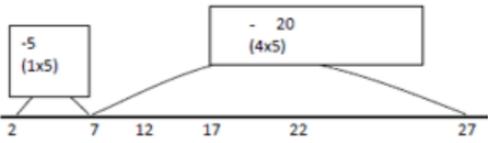
Stage 2	**15÷5=3 Count the number of jumps to find the answer.**
Division using repeated addition to count up to the total.	
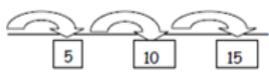
Stage 3	
Children should also move on to calculations involving remainders through repeated subtraction. Note: Many children continue to find repeated addition simpler. Make the link between subtraction and addition.	Repeated subtraction using a number line or bead bar **13÷3=4r1**
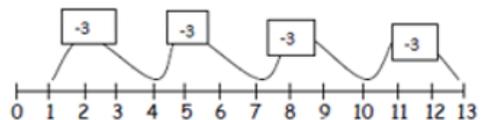
Stage 7	
Division with objects showing remainders and carrying TU ÷ U then HTU ÷ U	

Stage 8 Chunking **TU÷U then HTU ÷U** Then onto the vertical method. This is called chunking	$72 \div 3$ Answer: 24
Stage 9: Expanded division (long division) HTU÷U It is suggested that the multiples are written out first to avoid mistakes.	 64 r4 Step 1: Divide Step 2: Multiply Step 3: Subtract Step 4: Bring down

Stage 11: Long division **HTU ÷ U then HTU ÷ TU**	How many packs of 24 can we make from 560 biscuits? Using times tables knowledge, make sensible guesses. 24x20 =240 24x30=720 Start by taking estimate (24x20) away from the total Answer: 23 R 8 Conventionally, the long division method looks like this: Answer: 23 R 8
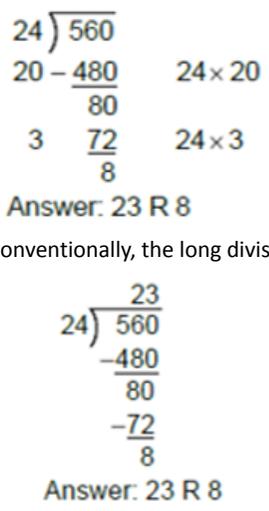
Stage 10: Short division HTU ÷ U **(with and without remainders)** **This is the National Curriculum formal written method.**	432 ÷ 5 becomes 8 6 r2 5 ⌐ 4 3³2 Answer: 86 remainder 2
Stage 9: Division with decimal remainders Continue with both expanded method and bus stop method to avoid calculation errors.	432 ÷ 5 becomes 0 8 6 . 4 5⌐4 3³2 .²0

Stage 12: Long division including decimal answers.
HTU ÷ U then HTU ÷ TU
 	
This is the National Curriculum formal written method.
	432 ÷ 15 becomes
	
	     2 8 . 8
	15 ⌐4 3 2 . 0
	    3 0 ↓
	   1 3 2
	   1 2 0 ↓
	   1 2 0
	   1 2 0
	      0
	
	Answer: 28.8
	
	This can also be calculated as a remainder (r 12) or as a fraction answer 4/5 (by simplifying 12/15).
	
	See NC guidance notes for examples)